SM223 – Calculus III with Optimization
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Lesson 19. Partial Derivatives, cont.

0 Warm up

Example 1. Find all the second partial derivatives of $f(x, y) = \ln(ax + by)$.

Example 2. Find all the second partial derivatives of $w(u, v) = 1 + \sqrt{1 + uv^2}$.

1 More examples

Example 3. The temperature at a point (x, y) on a flat metal plate is given by $T(x, y) = 60/(1 + x^2 + y^2)$, where T is measured in °C and x, y in meters. Find the rate of change in temperature with respect to distance at the point $(2, 1)$ in the x-direction and the y-direction.

Example 4. The average energy E (in kcal) neeeded for a lizard to walk or run a distance of 1 km has been modeled by the equation

$$E(m, v) = 2.65m^{0.66} + \frac{3.5m^{0.75}}{v}$$

where m is the body mass of the lizard (in grams) and v is its speed (in km/h). Calculate $E_m(400, 8)$ and $E_v(400, 8)$ and interpret your answers.

Example 5. Cobb and Douglas used the equation $P(L, K) = 1.01L^{0.75}K^{0.25}$ to model the productivity of the American economy from 1899 to 1922, where L is the amount of labor and K is the amount of capital.

a. Calculate P_L and P_K.

b. Find the rate of change in productivity with respect to labor and capital in the year 1899, when $L = 100$ and $K = 100$. Interpret the results.

c. Do the same for the year 1920, when $L = 194$ and $K = 407$.

d. In the year 1920, which would have benefited production more, an increase in capital investment or an increase in spending on labor?

Example 6. Consider the contour map of a function f given below. Are the following derivatives at the given point positive or negative?

a. f_x

b. f_y

c. f_{xx}

d. f_{yy}

e. f_{xy}